

2016 ATAR Math Exams

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Applications

150 marks total of which 107 from 1's and 2's (71%)

6 rounding instructions

Q4 – arithmetic is back!

Question 4

- (a) Given the sequence 256, 128, 64, 32, ...
 (ii) Deduce a rule for the n^{th} term of this sequence. H leaving your answer as a fraction.

$$256 \left(\frac{1}{2}\right)^{n-1}$$

$$256 \cdot \left(\frac{1}{2}\right)^{n-1}$$

ans | n=15

$$\frac{1}{64}$$

(3 marks)

- (b) Use the recursive definitions given to state the first **three** terms of each of the following sequences.

(ii) $T_{n+1} = 1.5T_n, T_2 = 7.5$

5

ans×1.5

5

(2 marks)

- (c) Consider the sequence 12, 7, 2, -3, ...

ans×1.5

7.5

11.25

By deducing a rule for the n^{th} term, or otherwise, determine which term of the sequence is -168. (3 marks)

$$12 + (n-1)(-5) = -168$$

$$-5 \cdot (n-1) + 12 = -168$$

solve (ans, n

{n=37}

Question 7

Julie buys a car with a purchase price of \$13 000. However, she has the car to depreciate in value. The value of the car after n years can be modelled by the recursive rule.

$$T_{n+1} = 0.85T_n, T_0 = 13\,000$$

- (a) Complete the table below to show the value of the car at the nearest dollar.

n	0	1	
Value of car after n years (\$)	13 000		

- (b) Use the information above to determine the rate of depreciation.

- (c) Determine a rule for the n^{th} term of the sequence of values for the car.

(9 marks)

Edit Graph

Recursive Explicit

☒ $a_{n+1} = 0.85 \cdot a_n$

$a_0 = 13000$

☐ $b_{n+1} = \square$

$b_0 = 0$

☐ $c_{n+1} = \square$

$c_0 = 0$

$a_{n+1} = 0.85 \cdot a_n$

n	a_n
0	13000
1	11050
2	9392.5
3	7983.6
4	6786.1
5	5768.2

7983.625

Deg Real

(d) Determine the value of Julie's car after eight years, correct to the nearest dollar. (2 marks)

(e) Julie decides that she will sell her car at the end of the year in which its value drops to half of the purchase price. After how many years should she sell her car? (2 marks)

n	a_n
0	13000
1	11050
2	9392.5
3	7983.6
4	6786.1
5	5768.2
6	4902.9
7	4167.5
8	3542.4
9	3011.0
10	2559.4

Question 8

(17 marks)

An experiment was conducted to determine whether there was any relationship between the maximum tidal current, in centimetres per second, and the tidal range, in metres, at a particular marine location. (The tidal range is the difference between the height of high tide and the height of low tide.) Readings were taken over a period of 12 days and the results are shown in the following table.

Tidal range	2.0	2.4	3.0	3.1	3.4	3.7	3.8	3.9	4.0	4.5	4.6	4.9
Maximum tidal current	15.2	22.0	25.2	33.0	33.1	34.2	51.0	42.3	45.0	50.7	61.0	59.2

Straightforward bivariate data question addressing many syllabus items.
Unusual for no variables to be defined anywhere in question.

Edit Action Interactive

$$\frac{210+230+100+90+160}{5}$$

158

$$5 \times 142 - 180 - 220 - 70 - 150$$

90

$$\text{ans} / 142$$

0.6338028169

Alg Decimal Real Deg M

Question 12

(11 marks)

Thomas has borrowed \$16 000 from a bank at a reducible interest rate of 18% per annum with interest accrued and repayments made monthly. Standard repayments are set at \$500 per month.

The table below shows the progress of the loan for the first six months rounded to the nearest cent.

Month	Amount owing at beginning of month	Interest for the month	Repayment
1	16 000.00	240.00	500.00
2	15 740.00	236.10	500.00
3	15 476.10	232.14	500.00
4	15 208.24	228.13	500.00
5	14 936.37	224.04	500.00
6	14 660.41	<i>A</i>	500.00

18/12

1.5

$14660.41 \times 1.5 / 100$

219.90615

$14660.41 + 219.91 - 500$

14380.32

(a) What is the monthly interest rate? (1 mark)

(b) Determine the values of *A* and *B*. (2 marks)

(c) Determine the length of time it will take Thomas to pay off the loan. (1 mark)

(d) Determine the total amount repaid on the loan. (3 marks)

(e) The bank suggests that Thomas could instead borrow the same amount at a rate of \$240 per month. Describe how this would affect the total amount he pays over the duration of the loan. (2 marks)

(f) After listening to advice, Thomas decides to pay off the loan completely in two years, making equal payments. Determine the amount of the monthly payment. (3 marks)

Compound Interest

N	43.921249
I%	18
PV	16000
PMT	-500
FV	0
P/Y	12
C/Y	12

N	Undefined
I%	18
PV	16000
PMT	-240
FV	0
P/Y	12
C/Y	12

N	43.921249
I%	18
PV	16000
PMT	-500
FV	0
P/Y	12
C/Y	12

Compound Interest

N	24
I%	18
PV	16000
PMT	-798.7856315
FV	0
P/Y	12
C/Y	12

Question 13**(6 marks)**

Simon has \$5000 that he wants to invest for a period of time without touching it.

- (a) If he chooses to invest this money in an account earning compound interest at the rate of 6.5% per annum, determine the:

(i) value of his investment after three years, if interest is paid annually. (1 mark)

(ii) time required for him to double his investment, if interest is paid monthly. (2 marks)

Edit Calc(1) Calc(2)	
Compound Interest	
N	3
I%	6.5
PV	-5000
PMT	0
FV	6039.748125
P/Y	1
C/Y	1

Edit Calc(1) Calc(2)	
Compound Interest	
N	128.3118949
I%	6.5
PV	-5000
PMT	0
FV	10000
P/Y	12
C/Y	12

- (b) Simon is currently deciding between two options and wishes to compare them.

Option A: Invest the \$5000 in an account earning compound interest at the rate of 5.5% per annum, with interest paid monthly.

Option B: Invest the \$5000 in an account earning compound interest at the rate of 5.4% per annum, with interest paid daily.

He decides to calculate the effective annual rate of interest for each option, in order to compare the possible investments. He determines that Option A has an effective annual rate of interest of 5.64%, correct to two decimal places.

Calculate the effective annual rate of interest for Option B and hence decide on the better option for Simon.

Edit Calc(1) Calc(2)	
Interest Conversion	
N	365
EFF	5.548038642
APR	5.4

(13 marks)

is planning to take an annuity from his pension fund. He sets 65th birthday with \$500 000 and he estimates the fund can % per year. He plans to start withdrawing an annuity of owing birthday.

relation to calculate the total s

s will Alex be able to receive

(iii) Assuming that all other conditions are the same, Alex decided to withdraw \$30 000 per year inste

Edit Calc(1) Calc(2)

Compound Interest

N	23.79132209
I%	6
PV	-500000
PMT	40000
FV	0
P/Y	1
C/Y	1

Edit Calc(1) Calc(2)

Compound Interest

N	Undefined
I%	6
PV	-500000
PMT	30000
FV	0
P/Y	1
C/Y	1

on July 1 2016 with a pri rate of 7.5% compounded n or on July 1, starting in 201

the fund after the annuity

und Interest

4

7.5

PV -850000

PMT 75000

FV 809531.4722

P/Y 1

C/Y 12

(iii) Calculate, to the annually, startin

bey could withdraw e. (2 marks)

Edit Calc(1) Calc(2)

Compound Interest

N	5
I%	9
PV	-809531.47
PMT	75000
FV	815197.7284
P/Y	1
C/Y	12

Edit Calc(1) Calc(2)

Compound Interest

N	1
I%	9
PV	-809531.47
PMT	75939.63577
FV	809531.47
P/Y	1
C/Y	12

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Specialist

150 marks total of which 20 from 1's and 2's (13%)

6 rounding instructions

Question 9

(5 marks)

Consider the integral $I = \int x \sqrt{(1+x)^n} dx$, where n is any positive integer.

Using the substitution $u = 1 + x$ and an appropriate anti-derivative, develop a simplified expression for I in terms of x and n .

$x \sqrt{(1+x)^n} \big|_{x=u-1}$
 $\sqrt{u^n} \cdot (u-1)$
 $\int \sqrt{u^n} \cdot (u-1) du$

$x \sqrt{(1+x)^n} \big|_{x=u-1}$
 $\sqrt{u^n} \cdot (u-1)$
 $\int \sqrt{u^n} \cdot (u-1) du$
 $\frac{-4 \cdot u^{\frac{n+4}{2}}}{(n+4) \cdot (n+2)} + \frac{2 \cdot u^{\frac{n+2}{2}} \cdot (u-1)}{n+2}$
 $\text{ans} \big|_{u=1+x}$
 $\frac{-4 \cdot (x+1)^{\frac{n+4}{2}}}{(n+4) \cdot (n+2)} + \frac{2 \cdot x \cdot (x+1)^{\frac{n+2}{2}}}{n+2}$
 $\text{simplify}(\text{ans})$
 $\frac{2 \cdot (x+1)^{\frac{n}{2}+1} \cdot (n \cdot x + 2 \cdot x - 2)}{(n+4) \cdot (n+2)}$

Question 11

(7 marks)

A lift goes up within a high rise building so that its velocity $v(t)$ is given by the graph shown below. The maximum velocity of the lift during its ascent is 1.2 ms^{-1} . For the first four seconds, the acceleration is given by $a(t) = kt$. For the final four seconds of its ascent, the lift decelerates at the same rate.

- (a) Show that the value of the constant $k = \frac{3}{20}$

$$\frac{3}{20} \times 2 \times 0.1$$

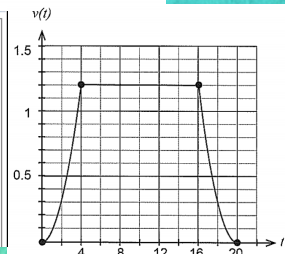
$$\int_0^4 \frac{3t^2}{20 \times 2} dt$$

$$\int_0^4 1 dv = \int_0^4 k \times t dt$$

$$\text{solve (ans } | v = 1.2 | t = 4, k$$

$$v = \frac{k \cdot t^2}{2}$$

$$\left\{ k = \frac{3}{20} \right\}$$



- (b) Using the increment $t = 2$ to $t = 2.1$ second

$$\text{ans} \times 2 + 1.2 \times 1.2$$

$$\frac{8}{5}$$

change in velocity v from

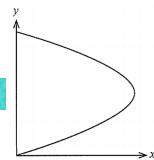
(2 marks)

- (c) Determine the total distance that the lift travels upwards during its ascent, correct to the nearest 0.1 m. (3 marks)

Question 13

(5 marks)

The graph of the curve $2x = \sin(y)$ is sketched for $0 \leq y \leq \pi$.



- (a) Determine the expression for $\frac{dy}{dx}$ in terms of y .

(2 marks)

- (b) Determine the area of the region bounded by the curve $2x = \sin(y)$ and the y axis.

(3 marks)

Edit Action Interactive

$2x = \sin(y)$

$2 \cdot x = \sin(y)$

impDiff (

$y' = \frac{2}{\cos(y)}$

$\int_0^\pi \frac{\sin(y)}{2} dy$

1

Question 14

(7 marks)

Consider the complex equation $z^4 = -16i$.(a) Solve the equation giving all solutions in the form $r \operatorname{cis} \theta$ where $-\pi < \theta \leq \pi$. (4 marks)

The first screenshot shows the equation $z^4 = -16i$ entered into the calculator. The second screenshot shows the solution for z in polar form: $z = 2 \cdot \left(\cos\left(\frac{7\pi}{8}\right) + i \sin\left(\frac{7\pi}{8}\right) \right)$. The third screenshot shows the conversion of the solution to polar form: $16 \cdot \left(\cos\left(\frac{-\pi}{2}\right) + i \sin\left(\frac{-\pi}{2}\right) \right)$.

Question 16

The screenshot shows the equation $x = 4 \cdot \cos\left(2 \cdot \cos^{-1}\left(\frac{y}{2}\right)\right)$ entered into the calculator. The result is $x = 2 \cdot y^2 - 4$.

(b) Determine the speed of the particle, correct to 0.01 cm per second, at the point where $x = -2$.

The screenshot shows the equation $4 \cos(2t) = -2$ entered into the calculator. The result is $t = \frac{\pi}{3}$.

The screenshot shows the calculation of the speed of the particle. The result is 16.81863357 .

(c) Write the expression, in terms of trigonometric functions, for the distance the particle will travel along its path in travelling from point A to point B. Do not evaluate this expression. (3 marks)

Edit Action Interactive

$x^2 + y^2 = 16$

$x^2 + y^2 = 16$

$y' = \frac{-x}{y}$

$y - (-2\sqrt{3}) = \frac{-2}{-2\sqrt{3}}(x - 2)$

$y + 2\sqrt{3} = \frac{\sqrt{3} \cdot (x - 2)}{3}$

$\sqrt{3} \cdot (y + 2\sqrt{3}) = x - 2$

$\sqrt{3} \cdot y + 6 = x - 2$

The region bounded by the arc AD , x axis.

(c) Determine the volume of the solid obtained by rotating the region about the x axis.

Edit Action Interactive

Define $f(x) = -\sqrt{16 - x^2}$

Define $g(x) = \frac{x - 8}{\sqrt{3}}$

$\int_{-4}^2 \pi f(x)^2 dx$

$\int_2^8 \pi g(x)^2 dx$

$72 \cdot \pi$

$24 \cdot \pi$

$72 \cdot \pi + 24 \cdot \pi$

$96 \cdot \pi$

301.5928947

Alg Standard Real Rad

being intersecting

in the form (3 marks)

(1 mark)

is rotated about the

est 0.01 cubic units. (4 marks)

Edit Action Interactive

$\frac{15}{\sqrt{50}}$

2.121320344

$\text{normCDF}(173, 177, s, 175)$

0.6542214138

$8960/50$

179.2

$\text{normCDF}(-\infty, 179.2, s, 175)$

0.9761425599

(b) total volume of water used is

Water is a scarce commodity and accuracy is required. The pool is topped up sample mean obtained is denoted by \bar{W} .

(c) If it is required that $P(a \leq \bar{W} \leq b) = 0.99$, then determine the values of correct to 0.1 litres.

(d) If the probability for the mean amount of water used differs from μ by less than five litres is 96%, find n , the number of waterings that need to be measured. (3 marks)

Edit Solve

Spec U34

Normal Cdf

Result

$n = 37.960961$

Left-Right=0

OK

Equation:

$n = \left(\frac{z \cdot \sigma}{e} \right)^2$

$n = 37.9609612912912$

$z = 2.05374891063182$

$\sigma = 15$

$e = 5$

Lower = $-9E+999$

Rad Real 1E-10

Q19 – Testing difference of means is NOT in our syllabus

A rival company called WollliWorks takes over the watering of the ornamental pool. Over 36 consecutive days, it was observed that the WollliWorks company used a total of 6.57 kilolitres. The standard deviation for the 36 days was also 15 litres.

A representative from the SavaDaWater company states that 'WollliWorks are using significantly more water than we did when we were filling this pool. They are wasting water'.

(e) Perform the calculations necessary to comment on this claim. (4 marks)

```
[6570/36, 15, 36, 0.95]⇒[m,
[182.5 15 36 0.95]
-invNormCDF(0, c, 1, 0)⇒z
1.959963985
s/√(n)⇒se
2.5
z×se⇒e
4.899909961
[m-e, m+e]
[177.60009 187.39991]
```

Question 20

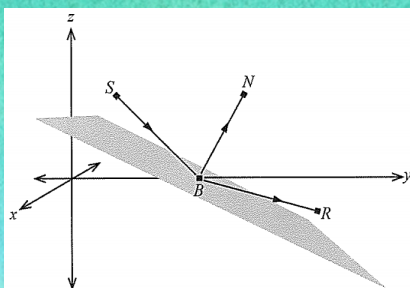
(7 marks)

A laser pointer at point S directs a highly focused beam of light towards a mirror. The beam bounces off the mirror at point B and is then reflected away from the mirror toward point R .

The mirror's surface is given by the equation $\vec{r} \cdot (\vec{j} + 2\vec{k}) = 9$ and the laser pointer is positioned at point S with position vector $-2\vec{i} + 3\vec{j} + 6\vec{k}$. The laser pointer is held so that the beam is pointed in the direction $\vec{d}_1 = \vec{i} + \vec{j} - \vec{k}$.

(a) Determine the position vector for point B .

(4 marks)



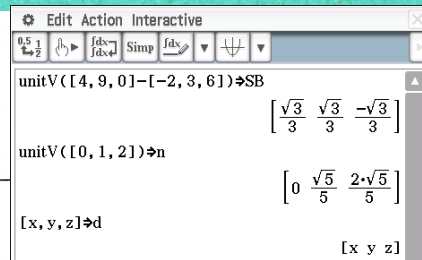
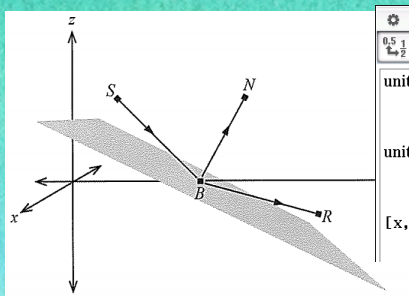
```
Edit Action Interactive
0.5 1 2
dotP([ -2+t, 3+t, 6-t ], [ 0, 1, 9 ]) = 9
-9*(t-6)+t+3=9
solve(ans, t)
{t=6}
[ -2+t, 3+t, 6-t ] | ans
[ 4, 9, 0 ]
```

The laser beam is reflected away from the mirror so that:

- the angle of the incoming beam \vec{SB} to the normal of the mirror is equal to the angle of the reflected beam \vec{BR} to the normal of the mirror i.e. $s\angle SBN = s\angle RBN$
- the incoming beam \vec{SB} , the normal of the mirror and the reflected beam are in one plane.

Let \hat{d}_2 = the unit vector in the direction of the reflected beam \vec{BR} i.e.

(b) Determine the unit vector \hat{d}_2 giving components correct to 0.0



$$\begin{aligned} \text{dotP}(\vec{SB}, \vec{n}) &= \text{dotP}(-\vec{n}, \vec{d}) \\ \frac{-\sqrt{15}}{15} &= \frac{-\sqrt{5} \cdot y}{5} - \frac{2 \cdot \sqrt{5} \cdot z}{5} \\ \text{crossP}(\vec{SB}, \vec{n}) &= \begin{bmatrix} \frac{\sqrt{15}}{5} & \frac{-2 \cdot \sqrt{15}}{15} & \frac{\sqrt{15}}{15} \end{bmatrix} \\ \text{crossP}(-\vec{n}, \vec{d}) &= \begin{bmatrix} \frac{2 \cdot \sqrt{5} \cdot y}{5} - \frac{\sqrt{5} \cdot z}{5} & \frac{-2 \cdot \sqrt{5} \cdot x}{5} & \frac{\sqrt{5} \cdot x}{5} \end{bmatrix} \\ \text{solve}\left(\frac{-2 \cdot \sqrt{5} \cdot x}{5} = \frac{-2 \cdot \sqrt{15}}{15}\right) &= \left\{x = \frac{\sqrt{3}}{3}\right\} \\ \left\{\begin{array}{l} \frac{-\sqrt{15}}{15} = \frac{-\sqrt{5} \cdot y}{5} - \frac{2 \cdot \sqrt{5} \cdot z}{5} \\ \frac{2 \cdot \sqrt{5} \cdot y}{5} - \frac{\sqrt{5} \cdot z}{5} = \frac{\sqrt{15}}{5} \end{array}\right\} &= y, z \\ \left\{y = \frac{7 \cdot \sqrt{3}}{15}, z = \frac{-\sqrt{3}}{15}\right\} & \end{aligned}$$

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Methods

150 marks total of which 58 from 1's and 2's (39%)

3 rounding instructions

Q4 & Q19 – poor match for syllabus dot points

Question 4

(8 marks)

The displacement x micrometres at time t seconds of a magnetic particle on a long straight superconductor is given by the rule $x = 5 \sin 3t$.

(a) Determine the velocity of the particle when $t = \frac{\pi}{2}$. (3 marks)

(b) Determine the rate of change of the velocity when $t = \frac{\pi}{2}$. (3 marks)

Let v = velocity of the particle at t seconds.

(c) Determine $\int_0^{\frac{\pi}{2}} \frac{dv}{dt} dt$. (2 marks)

Syllabus dot points

The second derivative and applications of differentiation

3.1.12 identify acceleration as the second derivative of position with respect to time

Applications of integration

3.2.21 determine displacement given velocity in linear motion problems

3.2.22 determine positions given linear acceleration and initial values of position and velocity.

Question 19**(8 marks)**

The displacement in centimetres of a particle from the point O in a

$$x(t) = \frac{1}{3} \left(\frac{t}{2} - 4 \right)^2 - 2 \text{ for } 0 \leq t \leq 10, \text{ where } t \text{ is measured in seconds.}$$

Calculate the:

- time(s) that the particle is at rest.
- displacement of the particle during the fifth second.
- maximum speed of the particle and the time when this occurs.
- total distance travelled in the first 10 seconds.

Calculator interface showing the derivative and integral of the displacement function $x(t) = \frac{1}{3} \left(\frac{t}{2} - 4 \right)^2 - 2$.

The derivative $\frac{d}{dt}(\text{ans})$ is calculated as $\frac{t-8}{6}$.

The integral $\int_4^5 \frac{t-8}{6} dt$ is calculated as $-\frac{7}{12}$.

The integral $\int_0^{10} \left| \frac{t-8}{6} \right| dt$ is calculated as $\frac{17}{3}$.

Question 9**(7 marks)**

Fermium-257

$P = P_0 e^{kt}$, where

k is a constant

The half-life of

(a) Determine

(b) How many

Equation:

$$P = P_0 \cdot e^{k \cdot t}$$

☐ $P = 50$

☐ $P_0 = 100$

☒ $k =$

☐ $t = 100.5$

Lower = $-9E+999$

Upper = $9E+999$

Deg Real $1E-10$

Calculator interface showing the result of the half-life calculation.

Result: $k = -6.897E-3$, Left-Right = 0.

Calculator interface showing the equation and variables for the half-life calculation.

Equation: $P = P_0 \cdot e^{k \cdot t}$

☐ $P = 50$

☐ $P_0 = 100$

☒ $k = -6.89698687124325E-3$

☐ $t = 100.5$

Lower = $-9E+999$

Upper = $9E+999$

Deg Real $1E-10$

ose decay rate ca

t is measured in

half of the original

significant figures.

ams of the subst

Equation:

$$P = P_0 \cdot e^{k \cdot t}$$

☐ $P = 5$

☐ $P_0 = 100$

☐ $k = -6.8969$

☒ $t =$

Lower = $-9E+999$

Upper = $9E+999$

Deg Real $1E-10$

Calculator interface showing the result of the half-life calculation.

Result: $t = 434.35377$, Left-Right = $2E-14$.

Calculator interface showing the equation and variables for the half-life calculation.

Equation: $P = P_0 \cdot e^{k \cdot t}$

☐ $P = 5$

☐ $P_0 = 100$

☐ $k = -6.89698687124325E-3$

☒ $t = 434.353773536176$

Lower = $-9E+999$

Upper = $9E+999$

Deg Real $1E-10$

t and

marks)

marks)

marks)

Question 10

A survey in Western Australia was conducted on the proportion of Year 12 students who use the Type A calculator. Out of 1450 Year 12 students, the survey found that 986 used the Type A calculator.

Determine the following.

- (a) A 90% confidence interval, to three decimal places, for the proportion of Australian Year 12 students who use the Type A calculator in calculating this interval?
- (b) The margin of error in this confidence interval.

Edit Action Interactive

[986, 1450, 0.90] → [N, n, c]

[986 1450 0.9]

invNormCDF(0, c, 1, 0) → z

1.644853627

N/n → p

0.68

$\sqrt{\frac{p \times (1-p)}{n}}$ → se

0.0122502639

z × se → E

0.020149891

[p-E, p+E]

[0.659850109 0.700149891]

fRound(ans, 4)

[0.6599 0.7001]

Alg Decimal Real Deg

(2 marks)

Type A.
calculator.

n
as made
(3 marks)

(2 marks)**Q10 & Q20 - Hypothesis testing is NOT in our syllabus**

Another three surveys of Year 12 students were conducted across Australia.

Edit Action Interactive

1772/3221

0.5501397082

1021/1566

0.6519795658

2203/3221

0.6839490841

□

- (d) Using the sample proportion and the sample size that will halve the margin of error for Year 12 students who use the Type A calculator.

Edit Solve

Methods Units 3&4

Normal CDF

Result

n=5800

Left-Right=0

OK

Equation:

$n = \frac{z^2 \cdot p \cdot (1-p)}{e^2}$

☒ n= 5800.0000000884

☐ z= 1.64485362695147

☐ p= 0.68

☐ e= 0.0100749455

Lower= -9E+999

Deg Real 1E-10

Edit Solve

Methods Units 3&4

Normal CDF

Result

n=5888.3104

Left-Right=0

OK

Equation:

$n = \frac{z^2 \cdot p \cdot (1-p)}{e^2}$

☒ n= 5888.3104

☐ z= 1.645

☐ p= 0.68

☐ e= 0.01

Lower= -9E+999

Deg Real 1E-10

Question 11

(3 marks)

The area of a triangle can be found by the formula: $Area = \frac{ab \sin C}{2}$.

Calculator interface showing the formula $Area = \frac{ab \sin C}{2}$ and the calculation for an equilateral triangle with side length 10 cm.

Input: $x \times x \times \sin(60)$

Result: $\frac{\sqrt{3} \cdot x^2}{4}$

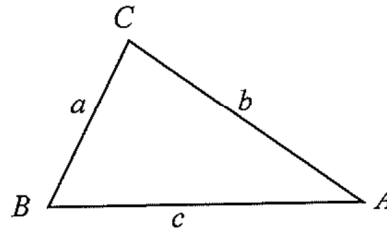
diff (

ans|x=10

ansx0.1

Result: $5 \cdot \sqrt{3}$

Result: $\frac{\sqrt{3}}{2}$



Using the formula, determine the approximate change in area of an equilateral triangle with side length 10 cm, when each side increases by 0.1 cm.

Question 12

(3 marks)

The Richter magnitude, M , of an earthquake is determined from the logarithm of the amplitude, A , of waves recorded by seismographs.

$$M = \log_{10} \frac{A}{A_0}, \text{ where } A_0 \text{ is a reference value.}$$

An earthquake in a town in New Zealand in November 2015 was estimated at 5.5 on the Richter scale, while the earthquake just north of Hayman Island measured 3.4 on the same scale. How many times larger was the amplitude of the waves in New Zealand compared to those at Hayman Island?

Calculator interface showing the calculation of the difference in Richter magnitude and the resulting ratio of amplitudes.

Input: $5.5 - 3.4$

Result: 2.1

Input: $10^{2.1}$

Result: 125.8925412

Input: \square

Question 13

(a) $x^2 \ln(x)$ (10 marks)

(b) $\text{diff} (x^2 \ln(x))$ (2 marks)

(c) $2 \cdot x \cdot \ln(x) + x$ (3 marks)

(d) (3 marks)

by the graph of $y = x^2 \ln x$, the x axis, $x = 1$ and $x = e$. (2 marks)

Question 14

The simulation results.

(a) Calculated $\sqrt{\frac{35 \times 25}{60 \times 60}}$ (2 marks)

(b) Determine the mean and standard deviation for the sample proportion of prime numbers in 60 tosses, using the results above. (2 marks)

(c) It has been decided to create a confidence interval for the proportion of prime numbers using the simulation results on page 8. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin of error. State this margin of error. (3 marks)

This simulation of 60 rolls of the die is performed another 200 times, with the proportion of prime numbers recorded each time and graphed.

(d) Comment briefly on the key features of this graph. (2 marks)

Question 16

An automated milk bottle labeller is uniformly to be

(a) Determine the probability that a machine c

(b) Calculate

A worker selects

(c) Determine the probability that a bottle containing

Edit Action Interactive

Define $f(x)=1/8$

done

$[247, 255] \rightarrow [a, b]$

$[247, 255]$

$\int_a^b f(x) dx \rightarrow \text{check_1}$

1

$\int_a^b x \cdot f(x) dx \rightarrow m$

251

$\int_a^b (x-m)^2 \cdot f(x) dx \rightarrow v$

$\frac{16}{3}$

$\sqrt{v} \rightarrow s$

$\frac{4\sqrt{3}}{3}$

approx (

2.309401077

Alg Standard Real Deg

(10 marks)

selected random

d amount.

ation of the amo

elt, one at a time.

the selection of 15 bottles before five bottles
 ant have been selected.

(3 marks)

Edit Action Interactive

$\text{binomialPDF}\left(4, 14, \frac{3}{8}\right)$

0.1800359861

$\text{ans} \times 3/8$

0.06751349479

□

Edit Action Interactive

$75+5$

80

$(22 \times 2)^2$

1936

$\begin{cases} 60=75a+b \\ 15=22a \end{cases} | a, b$

$\left\{ a=\frac{15}{22}, b=\frac{195}{22} \right\}$

approx (

$\{a=0.6818181818, b=8.8636\}$

□

Alg Standard Real Deg

(7 marks)

mination scores for all its Year 12 students taking Methods as a
 n percentage scores of all the Methods Year 12 students at the
 he mean was 75 with a standard deviation of 22.

(1 mark)**(2 marks)**

the results using the transformation $Y = aX + b$ where a and b
 d percentage scores. The aim is to change the mean to 60 and

a and b .

(4 marks)

Question 18**(6 marks)**

The waiting times at a Perth Airport departure lounge have been found to be normally distributed. It is observed that passengers wait for less than 55 minutes, 5% of the time, while there is a 13% chance that the waiting times will be greater than 100 minutes.

- (a) Determine the mean and standard deviation for the waiting times at Perth Airport departure lounge. **(5 marks)**

- (b) Determine the probability that the waiting times are between 75 and 90 minutes. **(1 mark)**

```

Edit Action Interactive
invNormCDF("L", 0.05, 1, 0)
-1.644853627
invNormCDF("R", 0.13, 1, 0)
1.126391129
{
  55-x = -1.644853627
  100-x = 1.126391129
}
{x=81.70944638, y=16.2381904}
normCDF(75, 90, 16.24, 81.71)
0.3553998348

```

Question 20

How is each box filled? 24

(14 marks)

A chocolate factory produces chocolates of which 80% contains exactly 30 pieces.

- (a) Identify the probability distribution of X = the number of chocolates in a randomly selected box and also give the mean and standard deviation. **(3 marks)**

- (b) Determine the probability, to three decimal places, that there are at least 27 pink chocolates in a randomly selected box. **(3 marks)**

Quality Control collects samples sizes of 20 boxes and counts the total.
 ???

- (c) Determine a 95% confidence interval for the proportion of pink chocolates in 20 boxes, using the assumption that 80% of chocolates in the

```

Edit Action Interactive
30*0.8
24
sqrt(30*0.8*0.2)
2.19089023
binomialCDF(27, 30, 0.8)
0.1227108064

```

Lower	0.7679939
Upper	0.8320061
\hat{p}	0.8
n	600

<< Back ☐ Help

OnePropZInt

Edit Action Interactive
 binomialPDF(1, 3, 0.05)
 0.13

(e) Using
 num
 Quality Co
 Samp
 Num
 of pin
 chocola
 (f) De

4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p

Unit 4 Overview: Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In the Mathematics Methods ATAR course, statistical inference is restricted to estimating proportions in two-outcome populations.

Question 21**Related Rates!!! Why???****(6 marks)**

A lighthouse is situated 12 km away from the shoreline, opposite point X as seen in the diagram below. A long brick wall is placed along the shoreline and at night the light from the lighthouse can be seen moving along this wall.

Let y = displacement of light on the wall from point X and θ = angle of the rotating light from the lighthouse.

The light is revolving anticlockwise at a uniform rate of three revolutions per minute

($\frac{d\theta}{dt} = 6\pi$ radians/minute).

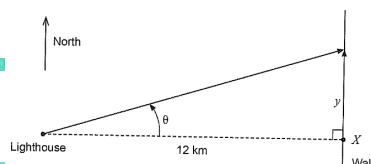
(a) Show that $\frac{dy}{d\theta} = \frac{12}{\cos^2 \theta}$.

(3 marks)

(b) Determine the velocity, in kilometres per minute, of the light on the wall when the light is 5 km north of point X .

(3 marks)

(Hint: $\frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$)



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