MATHEMATICS APPLICATIONS **2** **CALCULATOR-FREE**

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	9	9	100	94	65
				Total	100

150 147

eActivities used and pdf of this document are free downloads from HELPSHEETS page at www.charliewatson.com/atar/

- 97 out of 147 marks (66%) from question parts worth 1 or 2.
- Just 3 rounding instructions this year.

Fake annuity question...

The rise and rise of gobbledygook...

Q1

Section One: Calculator-free **35% (53 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

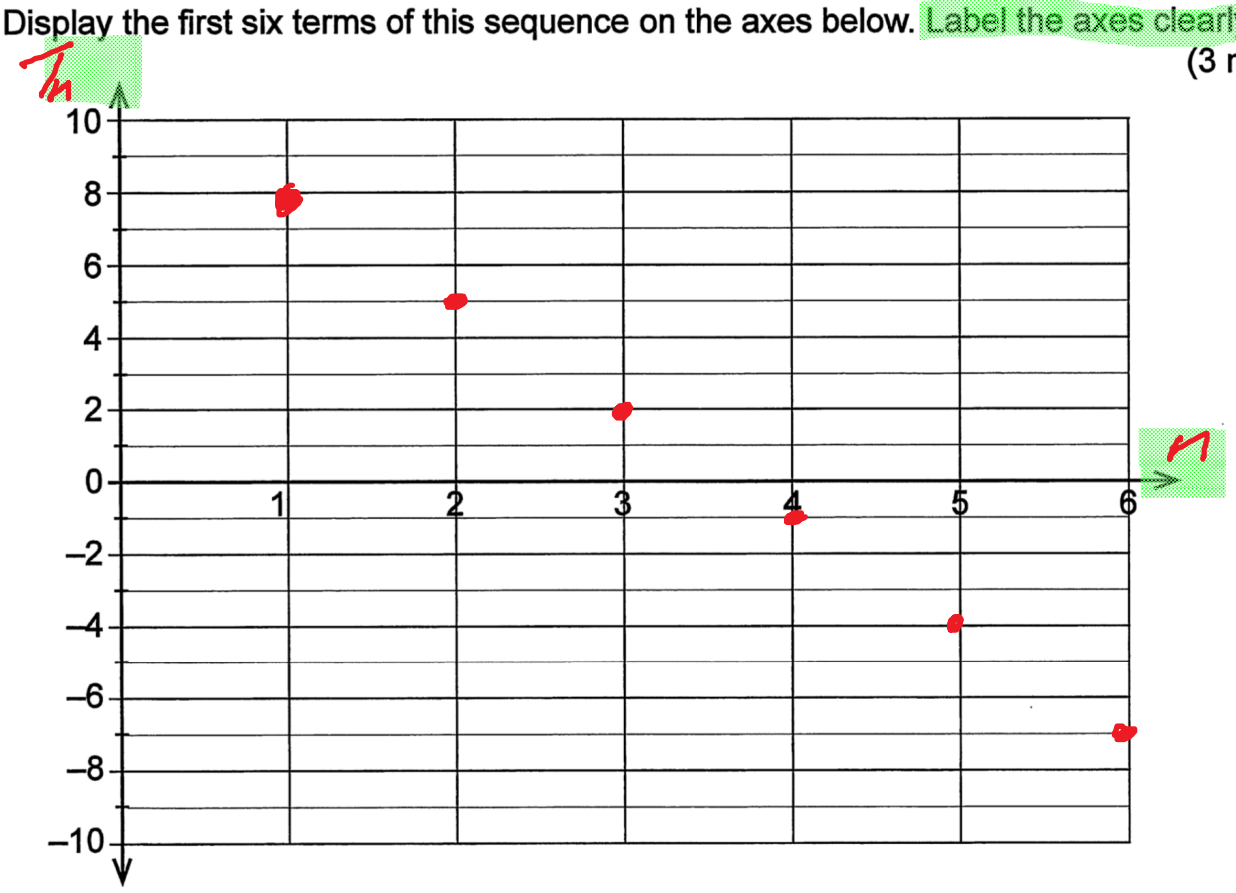
Working time: 50 minutes.

Question 1 **(8 marks)**

Consider the following recurrence relation:

$$T_{n+1} = T_n - 3,\ T_3 = 2.$$

(a) Display the first six terms of this sequence on the axes below. Label the axes clearly. (3 marks)



(b) (i) Deduce a rule for the n^{th} term of this sequence. (2 marks)

$$T_n = 8 + (n-1)(-3)$$

(ii) Hence, determine the first term in the sequence which is less than –500. (3 marks)

$$8 + (n-1)(-3) = -500$$

$$n - 1 = \frac{-508}{-3} = 169\tfrac{1}{3}n$$

$$n = 170\tfrac{1}{3} \Rightarrow T_{171}$$

Q2a

Question 2 **(7 marks**

A supermarket provides a delivery service to its customers. This morning, there are four deliveries (1, 2, 3 and 4) to be made. Each of four drivers, John, Kerry, Liam and Max, is available to do one of the deliveries.

The table below shows the time, in minutes, that each driver would take to complete each of the four deliveries.

Table 1

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	35	31	41	36
	2	25	26	33	36
	3	32	28	25	24
	4	27	30	31	28

The store manager will allocate the deliveries so that the total delivery time is at a minimum. He decides to use the Hungarian algorithm to determine the allocation of deliveries to the drivers.

His first step is to subtract the minimum entry in each row from each element, ensuring that eac row contains at least one zero.

Table 2

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	4	0	10	5
	2	0	1		11
	3	8	4	1	0
	4	0	3	4	1

(a) What is the number missing from the shaded cell? 8 (1 mark

The second step is to ensure that all columns contain at least one zero. The numbers that resul from this step are shown in the table below.

Table 3

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	4	0	9	5
	2	0	1	7	11
	3	8	4	0	0
	4	0	3	3	1

Q2b

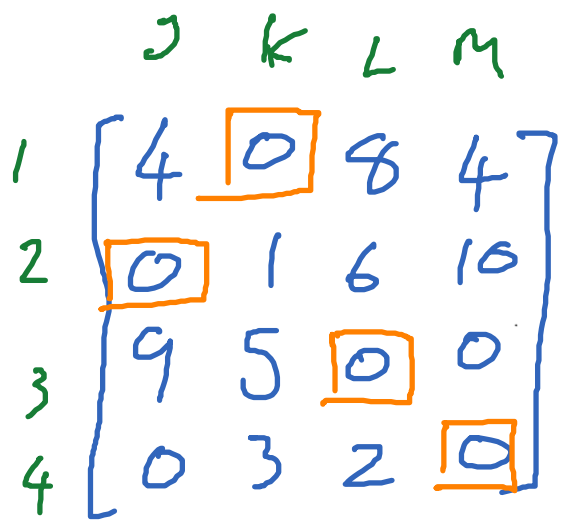
(b) The smallest number of horizontal and vertical lines that can be drawn to cover all the zeros is three.

(i) Draw in these lines on **Table 3** on the previous page. (2 ways) (1 mark)

(ii) State why an allocation of delivery drivers cannot be made yet. (1 mark)

Hmmm, I think we can, so what to say!!??
Algorithm is not yet complete…
Smallest number of lines must be FOUR…
Minimum assignment not yet possible…

(c) Continue the steps of the Hungarian algorithm to determine the optimum allocation of deliveries to the drivers. Complete the table at the bottom of the page and state the minimum total delivery time. (4 marks)



31 + 25 + 25 + 28 = 109

Delivery Driver	John	Kerry	Liam	Max
Delivery	2	1	3	4

Minimum total delivery time 109 minutes

Q3

Question 3 **(11 marks**

(a) A planar graph has five faces and five vertices, A, B, C, D and E.

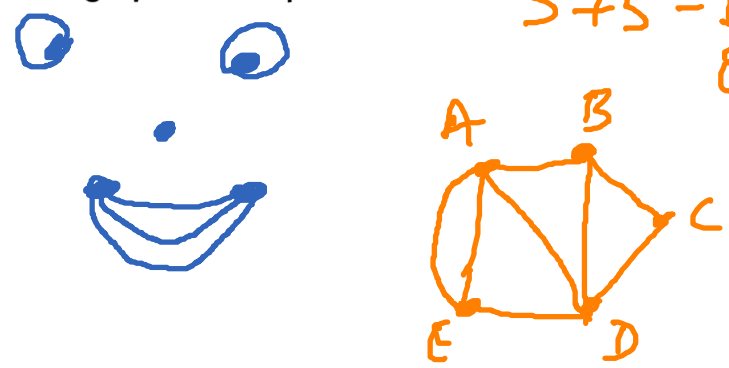
(i) Determine the number of edges for this graph. (2 marks

4, 5, 6, 7, ... ∞

(ii) Draw the planar graph in the space below. (2 marks

$5+5-E=2$

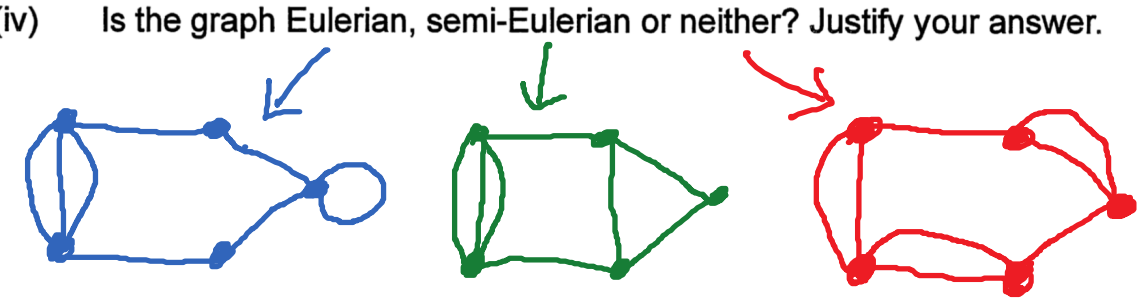
$E=8$



(iii) Determine a Hamiltonian cycle for the graph, giving your answer as a sequence o vertices. (1 mark

ABCDEA

(iv) Is the graph Eulerian, semi-Eulerian or neither? Justify your answer. (2 marks



(b) (i) A simple connected graph contains five vertices. Determine the minimum and the maximum number of edges it contains. (2 marks

Min = 4

$\text{Max} = \frac{5 \times 4}{2} = 10$

(ii) A simple connected graph contains n vertices. Determine the minimum number of edges it contains. (1 mark

$n-1$

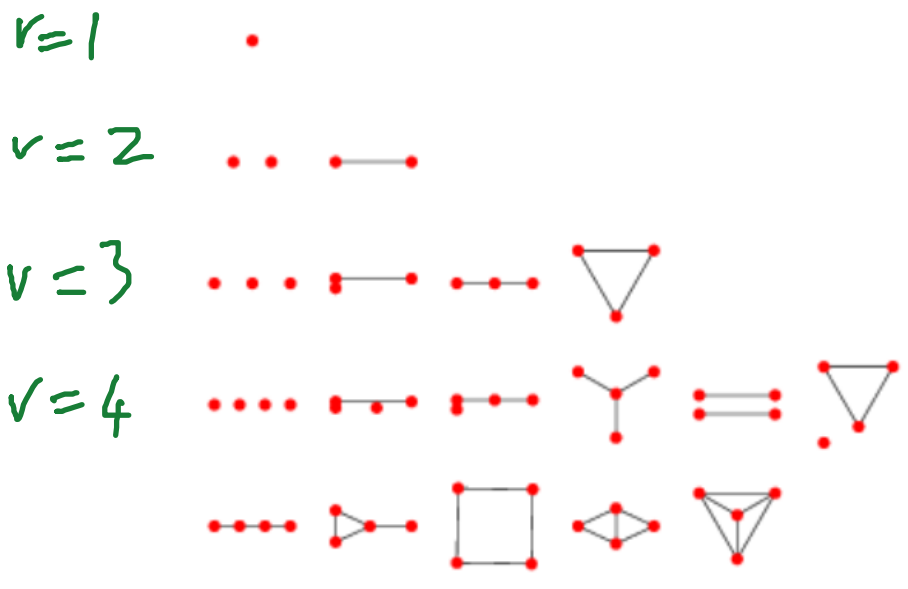
(iii) What name is given to the simple connected graph with the maximum number of edges possible? (1 mark

COMPLETE

Wolfram

Planar Graph

DOWNLOAD Wolfram Notebook | EXPLORE THIS TOPIC IN The *MathWorld* Classroom | CONTRIBUTE To this Entry



Glossary

Eulerian graph	A connected graph is Eulerian if it has a closed trail (starts and ends at the same vertex), that is, includes every edge and once only; such a trail is called an Eulerian trail. An Eulerian trail may include repeated vertices. A connected graph is semi-Eulerian if there is an open trail that includes every edge once only.

2016 WACE Q5c marking key

(c) One of the planar graphs is semi-Eulerian. State which graph it is, giving a reason for your choice. (2 marks)

Solution
graph (ii) is semi-Eulerian since it has exactly two odd vertices
Specific behaviours
✓ identifies correct graph ✓ states correct reason

To identify whether such a trail exists, we can **use** the degree of the vertices.

Question 4 **(5 marks)**

Ryan was keen to compare interest rates offered by different banks, so he decided to construct a table showing the effective annual rates of interest (%). Part of his table is shown below.

	Rate of interest (p.a.)				
Compounding period	**4%**	**4.5%**	**5%**	**5.5%**	**6%**
Quarterly	4.060	4.577	5.095	5.614	6.136
Monthly	4.074	4.594	5.116	5.641	6.168
Daily	4.081	4.602	5.127	5.654	6.183

(a) Ryan wants to borrow $5000 to purchase a second-hand car. A bank offers to lend him the money at the rate of 6% p.a. for one year. He plans to pay off the entire loan (including the interest) at the end of the year. Which compounding period should he sign up for? Justify your decision. (2 marks)

Quarterly - lowest rate, to minimise interest.

(b) Ryan is curious to know how much interest he would earn by investing $100 for a year, earning 4% p.a. with interest compounded quarterly. Determine the interest he would earn. (1 mark)

$$4.060 \times 100 \div 100 = \$4.06$$

(c) Ryan's sister has $3000 to invest for a year. She has been offered a rate of 5% p.a., with interest compounded daily. Determine the value of her investment at the end of the year. (2 marks)

$$5.127 \times \frac{3000}{100} = 51.27 \times 3 = \$153.81$$

Question 5 **(9 marks**

A group of university students was asked the question 'Does full attendance at school lead to an improved examination result?'

The results are summarised below.

	Agree	Disagree	Undecided
Male under 20 years	8	22	6
Female under 20 years	6	20	8
Male 20 to 25 years	26	7	3
Female 20 to 25 years	30	9	5
Male over 25 years	24	3	2
Female over 25 years	18	2	1

(a) Complete the two-way table below. (2 marks

	Agree	Disagree	Undecided	Total
Under 20	14	42	14	70
20–25	56	16	8	80
Over 25	42	5	3	50
	112	63	25	

(b) State the explanatory variable for these data. (1 mark

(c) The incomplete table below shows row percentages.

	Percentages		
	Agree	Disagree	Undecided
Under 20	20	60	20
20–25	70	20	10
Over 25	84	10	6

100 100 100

(i) Show how the value of 20% was calculated. (2 marks)

$56 + 16 + 8 = 80$

$16 \div 80 \times 100 = 20\%$

	Column N Percentages		
	Agree	Disagree	Undecided
Under 20	13	67	56
20–25	50	25	32
Over 25	37	8	12
	100	100	100

(ii) Complete the table. $\frac{14}{70} \times 100$, etc, etc! (2 marks)

(d) Use the data to determine one association between the variables. Describe the association and explain your reasoning. (2 marks)

An association between age group and level of agreement with the statement exists.

As age group increases, so does the percentage of those who agree with the statement increase.

Syllabus

Identifying and describing associations between two categorical variables

3.1.2 construct two-way frequency tables and determine the associated row and column sums and percentages

3.1.3 use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association

3.1.4 describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data

Association	Association is a general term used to describe the relationship between two (or more) variables. For example, is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and gender (male, female)? Is there an association between a person's height and foot length? The term association is often used interchangeably with the term correlation. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.

We want to determine if an association (relationship) exists between x and y. That is, are the two variables dependent?

Saying that two variables are associated means that knowing the value of one variable provides information about the value of the other variable.

Saying that two variables are not associated means that knowing the value of one variable provides no information about the value of the other variable.

Formal test for association is chi-squared … (doesn't indicate direction)

2016 Q6

Question 6 **(7 marks)**

Before a fitness campaign at a high school started, 50 students were chosen at random from each year group and asked the following questions:

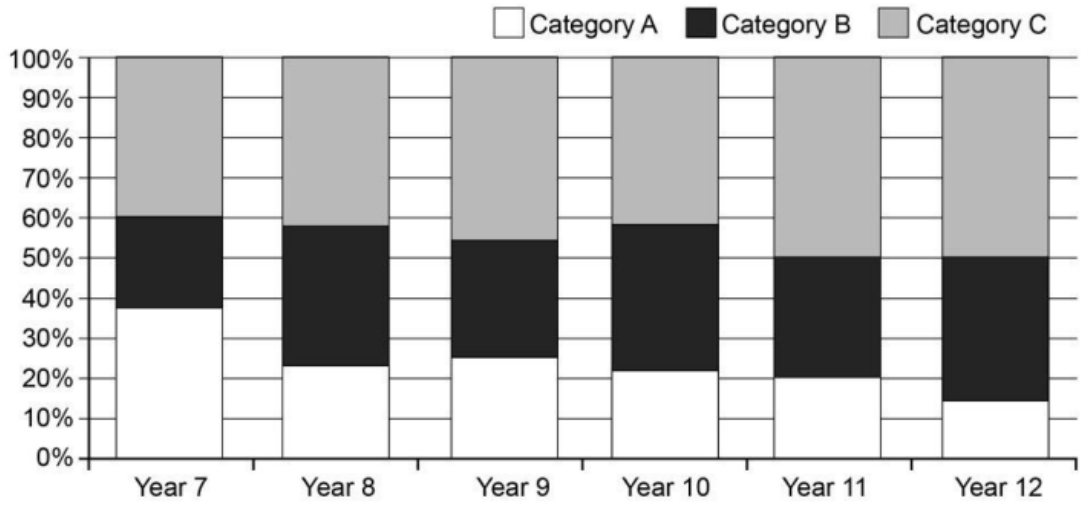
Question 1: Which one of the following modes of transport do you use to travel to and from school?

Category A: walking/cycling
Category B: public transport
Category C: private car

Question 2: Which year group are you in?

The campaign organisers wished to determine whether age group affected the students' likelihood of walking/cycling to and from school.

The data given in the table for part (a) have been displayed as a divided column graph below.



(c) Using the graph above or another method, comment on:

(i) the association between 'Year group' and 'Category A'. (1 mark)

Solution
Generally as the students get older the percentage of students using Category A as a mode of transport decreases.
Specific behaviours
✓ states correct association between year group and category A

(ii) the association between 'Year group' and 'Category C'. (1 mark)

Solution
Generally as the students get older the percentage of students using Category C as a mode of transport increases.
Specific behaviours
✓ states correct association between year group and category C

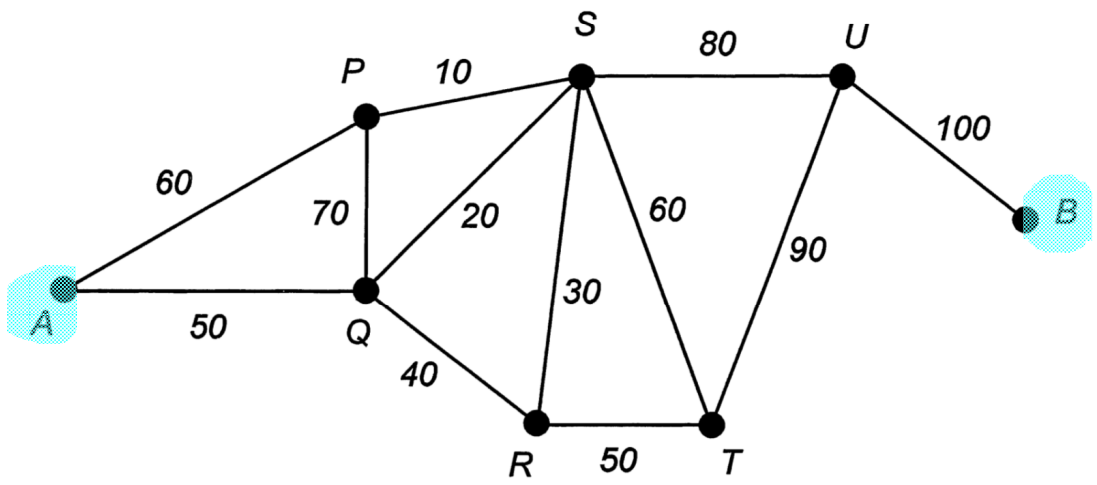
(iii) the association between 'Category A' and 'Category B and C combined'. (1 mark)

Solution
There are less students who walk or cycle to and from school than those who use motorised transport. **or** As category A increases, Category B and C decrease.
Specific behaviours
✓ states correct association between Category A and Category B and C combined

Q6a

Question 6 **(7 marks)**

(a) In the network below, the nodes represent towns and the numbers on the arcs represent the time taken (in minutes) to travel between them.

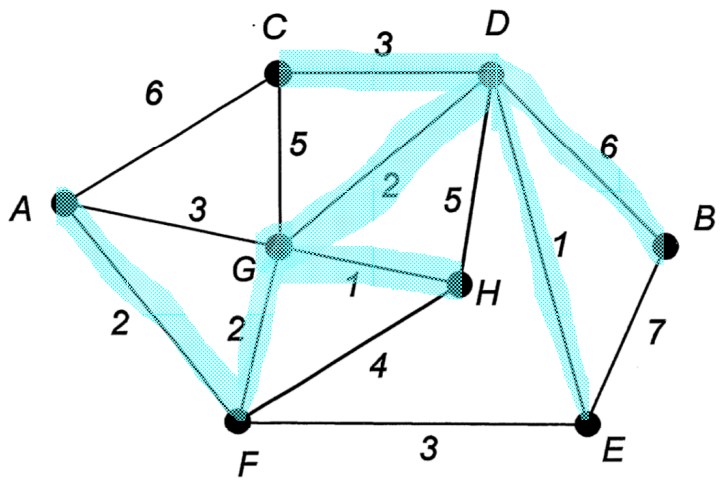


A driver leaves Town A and must deliver goods to all the other towns in the shortest time, finishing at Town B. Determine this shortest time. (A town may be visited more than once). (3 marks)

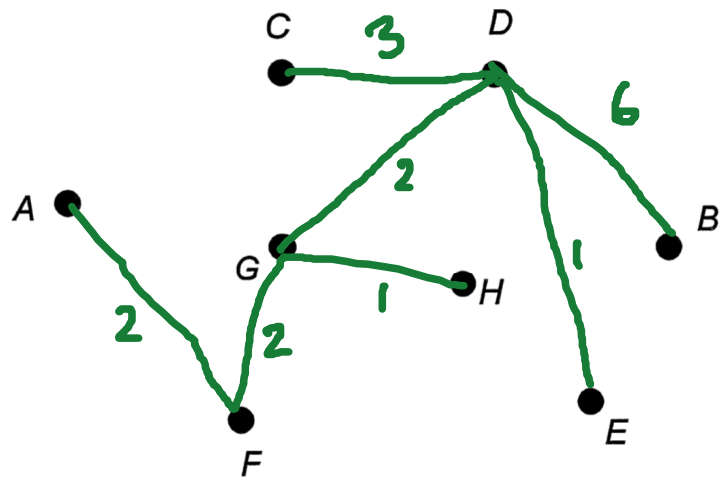
AQSPSRTUB=50+20+10+10+30+50+90+100=360
AQRTSPSUB=50+40+50+60+10+10+80+100=400
AQPSRTUB=50+70+10+30+50+90+100=400
APSQRTUB=60+10+20+40+50+90+100=370

Q6b

(b) The network below shows the distances (in metres) between stations for a model railway track system.



(i) Determine the minimal spanning tree for the network and draw this tree on the diagram below. (3 marks)

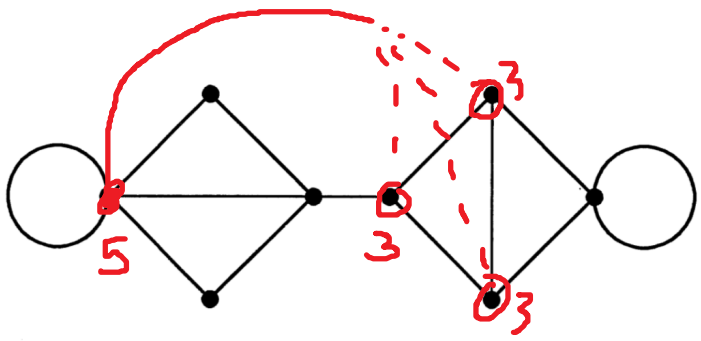


(ii) State the length of the minimal spanning tree. 17m (1 mark)

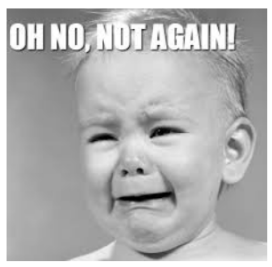
Q7

Question 7 **(6 marks)**

(a) The graph below shows the paths connecting the exhibits at a zoo.



(i) Explain why the graph is not semi-Eulerian. (1 mark)

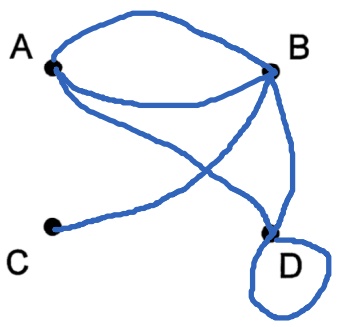


It does not contain an open trail
that includes every edge once only.

(ii) Draw one edge on the graph so that it becomes semi-Eulerian and does not contain a bridge. (2 marks)

(b) The adjacency matrix Q represents the raised paths connecting the observation platforms in the safari section at the zoo. Draw a planar graph for the adjacency matrix. (3 marks)

$$Q = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$



Q8

Section Two: Calculator-assumed **65% (94 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(6 marks)**

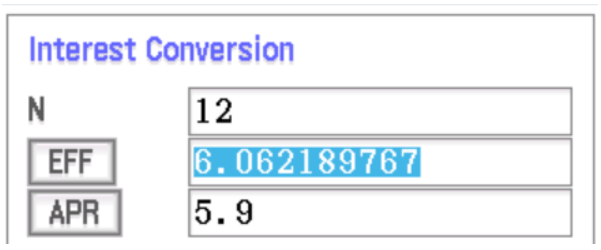
Ming, a former high school student and now a successful business owner, wishes to set up a perpetuity of $6000 per year to be paid to a deserving student from her school. The perpetuity is to be paid at the start of the year in one single payment.

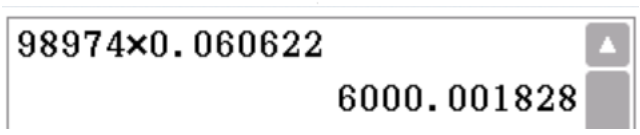
(a) A financial institution has agreed to maintain an account for this perpetuity paying a fixed rate of 5.9% p.a. compounded monthly.

Show that an amount of $98 974, to the nearest dollar, is required to maintain this perpetuity. (3 marks)

Effective rate is $6.0622\%\ pa$

$\$98974 \times 0.060622 = \$6000.00$

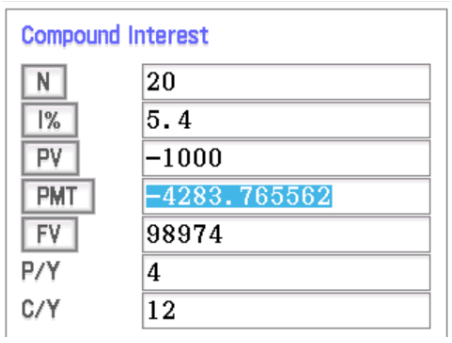




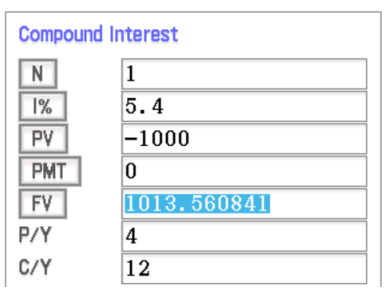
(b) Ming allows herself five years to accumulate the required $98 974 by making regular quarterly payments into an account paying 5.4% p.a. compounded monthly.

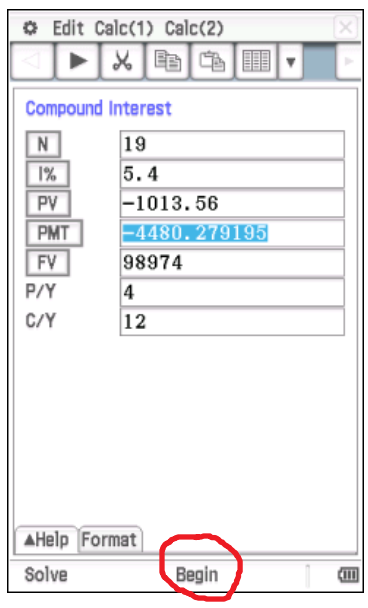
Determine the quarterly payment needed to reach the required amount after five years if Ming starts the account with an initial deposit of $1000. (3 marks)

End of qtr
$4 \times 5 = 20$ deposits
Deposit is $4283.77



Start of qtr
19 deposits
Deposit is $4480.28





Syllabus…

Topic 4.2: Loans, investments and annuities (20 hours)

Compound interest loans and investments

4.2.1 use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment

4.2.2 calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly

4.2.3 with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans, investments and depreciating assets

Reducing balance loans (compound interest loans with periodic repayments)

4.2.4 use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan

4.2.5 with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans

Annuities and perpetuities (compound interest investments with periodic payments made from the investment)

4.2.6 use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity

4.2.7 with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case)

Q9a

Question 9 **(13 marks**

The World Health Organisation produces tables showing Child Growth Standards. The median lengths (cm) for girls at various times during the first five years of life are shown below.

Age (months) x	0	3	12	21	27	42	48	60
Median length (cm) y	49.1	59.8	74.0	83.7	88.3	99.0	102.7	109.4
Predicted length (cm)	58.2	61.0	69.5	77.9	*A*	97.7	*B*	114.7
Residual	–9.1	–1.2	4.5	5.8	4.7	1.3	*C*	*D*

(a) (i) Determine the equation of the least-squares line for predicting the median length from a girl's age. (1 mark

$\hat{y} = 0.9424x + 58.1594$

(ii) Use the equation from (a)(i) to determine the predicted median lengths ***A*** and ***B*** in the above table.

A = 83.6 cm

B = 103.4 cm (2 marks

(iii) What increase in median length can be expected for each additional year? (1 mark

$0.9424 \times 12 = 11.3$ cm

(iv) Given that the correlation coefficient is 0.97, describe the association between age and median length in terms of its direction and strength. (2 marks

Positive and strong

(v) What percentage of the variation in the median length can be explained by the variation in age? (1 mark

$0.97^2 = 0.9397 \approx 94\%$

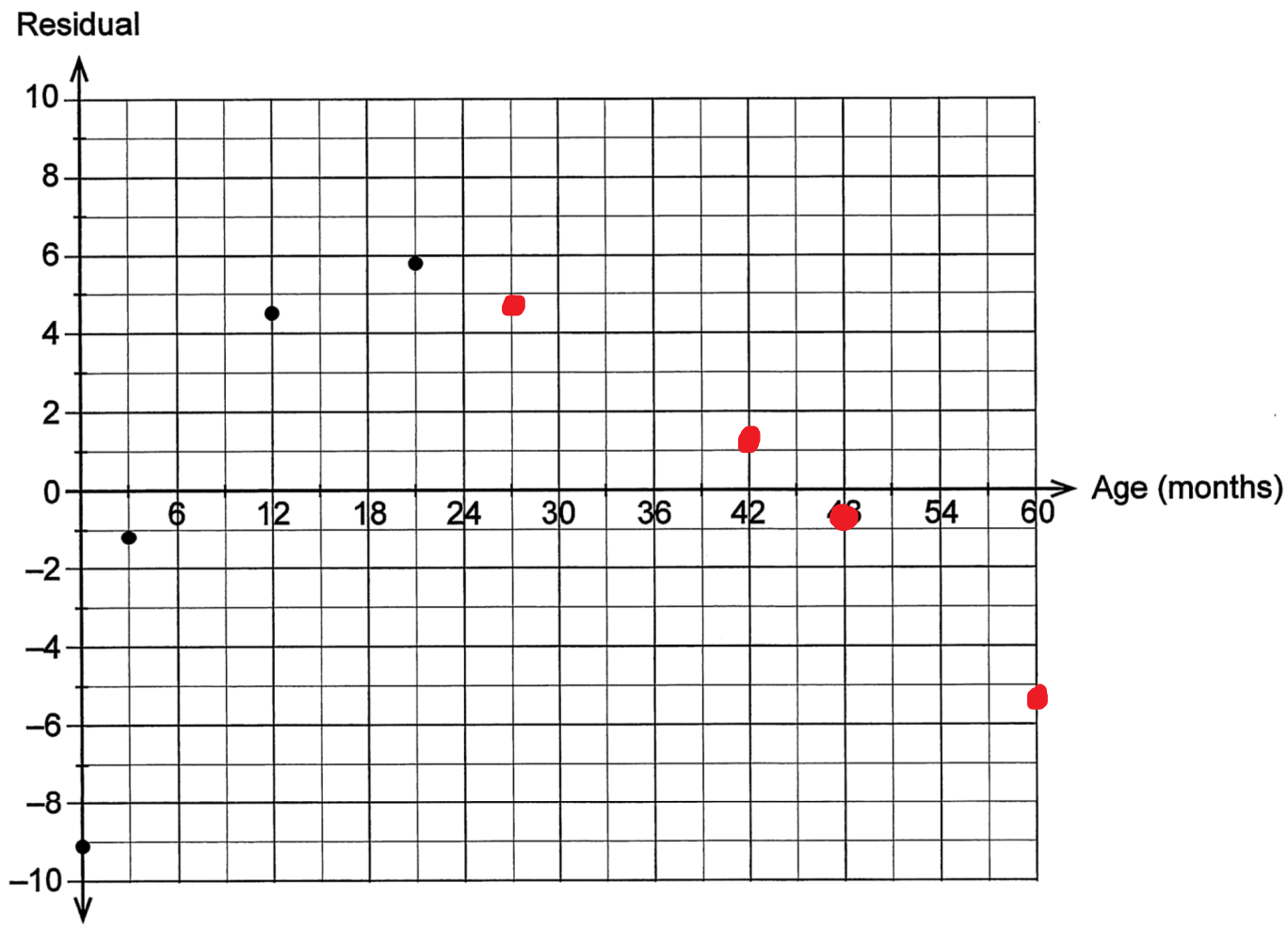
Q9b

(b) (i) Determine the residuals C and D in the table.

C = -0.7 cm

D = -5.3 cm (2 marks)

(ii) Hence, complete the scattergraph of the residuals against age on the axes below by plotting the last four residual values. (2 marks)



(iii) Use the residual plot to assess the appropriateness of fitting a linear model to the data. (2 marks)

Not appropriate as a pattern is clearly evident in the residual plot.

Q10

Question 10 **(12 marks)**

In a laboratory experiment, the population of a particular bacteria began with 400 present. The bacteria grew at a rate of 35% each week, where P is the number of bacteria and t is the numbe of weeks from the start of the experiment.

(a) Four possible equations were produced to model this experiment:

$P = 400(1.35)^t$ ✓

$P = 400(0.35)^t$ ✗

$P = 540(1.35)^{t-1}$ ✓

$P = 540(1.35)^{t+1}$. ✗

$a_nE=400\cdot1.35\text{^}n$	
n	a_nE
0	400
1	540
2	729
3	984.15
4	1328.6
5	1793.6
6	2421.4
7	3268.9
8	4413.0

Circle the correct equation(s). **(2 marks)**

(b) Calculate the population of bacteria after three weeks. **(1 mark)**

$400(1.35)^3 = 984$

```
400(1.35)^3
                    984.15
```

(c) During which week did the population of bacteria first reach 1800? **(2 marks)**

$t = 5, P = 1794$
$t = 6, P = 2421$, so during the 6th week.

(d) After eight weeks the growth rate slowed to 20% each week. How many weeks in total did it take for the population of bacteria to reach 15 812? **(3 marks)**

$8 + 7 = 15$ weeks

(e) What constant weekly growth rate would produce the same change in population from 400 to 15 812 in the same time as found in part (d)? **(2 marks)**

27.78%

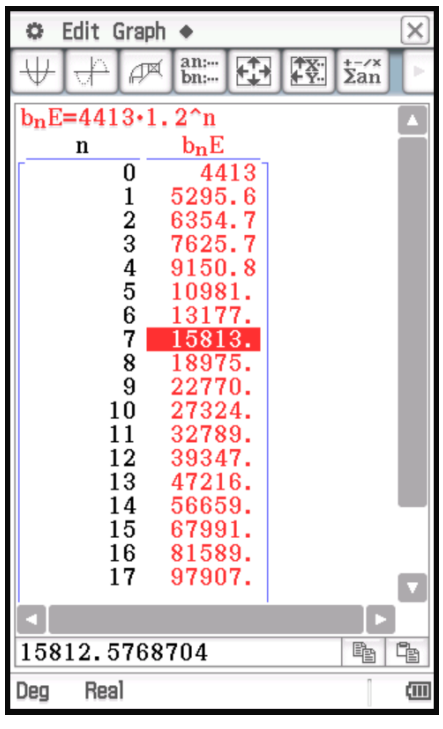
```
solve(400(x)^15=15812
                {x=1.277796775}
```

(f) Once the bacteria population reached 15 812 it began to die out at a rate of 250 each day. Approximately how many weeks did it take for the bacteria to die out completely? **(2 marks)**

9 weeks

```
15812/250
                    63.248
ans/7
                    9.035428571
```

Q10 CP


Edit Graph
bnE=4413·1.2^n
n
bnE
0 4413
1 5295.6
2 6354.7
3 7625.7
4 9150.8
5 10981.
6 13177.
7 15813.
8 18975.
9 22770.
10 27324.
11 32789.
12 39347.
13 47216.
14 56659.
15 67991.
16 81589.
17 97907.
15812.5768704
Deg
Real

Syllabus…

The geometric sequence

3.2.5 use recursion to generate a geometric sequence

3.2.6 display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations

3.2.7 deduce a rule for the n^{th} term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions

3.2.8 use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay

Algebraically too?

Unit description

This unit has three topics: 'Bivariate data analysis', 'Growth and decay in sequences', and 'Graphs and networks'.

'Bivariate data analysis' introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

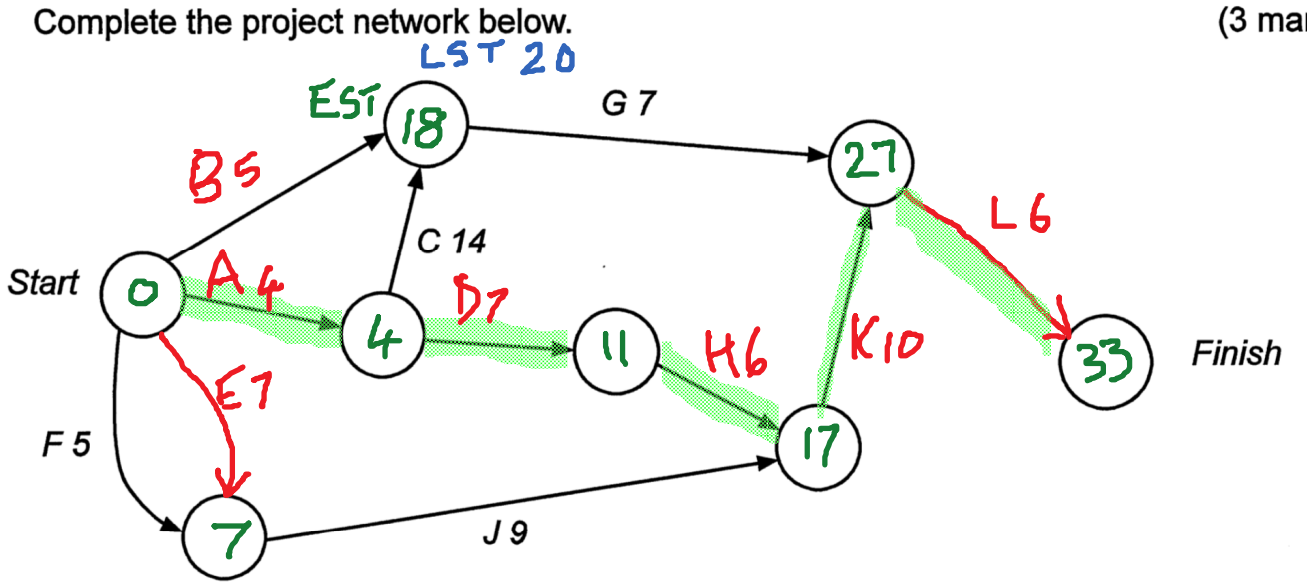
'Growth and decay in sequences' employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

Question 11 **(11 marks)**

The following table, consisting of 11 activities, contains information for a project in a small manufacturing company.

Activity	Immediate Predecessors	Time (hours)
A	–	4
B	–	5
C	A	14
D	A	7
E	–	7
F	–	5
G	B, C	7
H	D	6
J	E, F	9
K	H, J	10
L	G, K	6

(a) Complete the project network below. (3 marks)



(b) State the critical path and the minimum completion time for this network. (2 marks)

CP: ADHKL, MCT: 33 hours.

(c) Determine the float time, earliest starting time, and latest starting time for Activity G. (3 marks)

EST: 18 h, LST: 20 h, Float: 2 h.

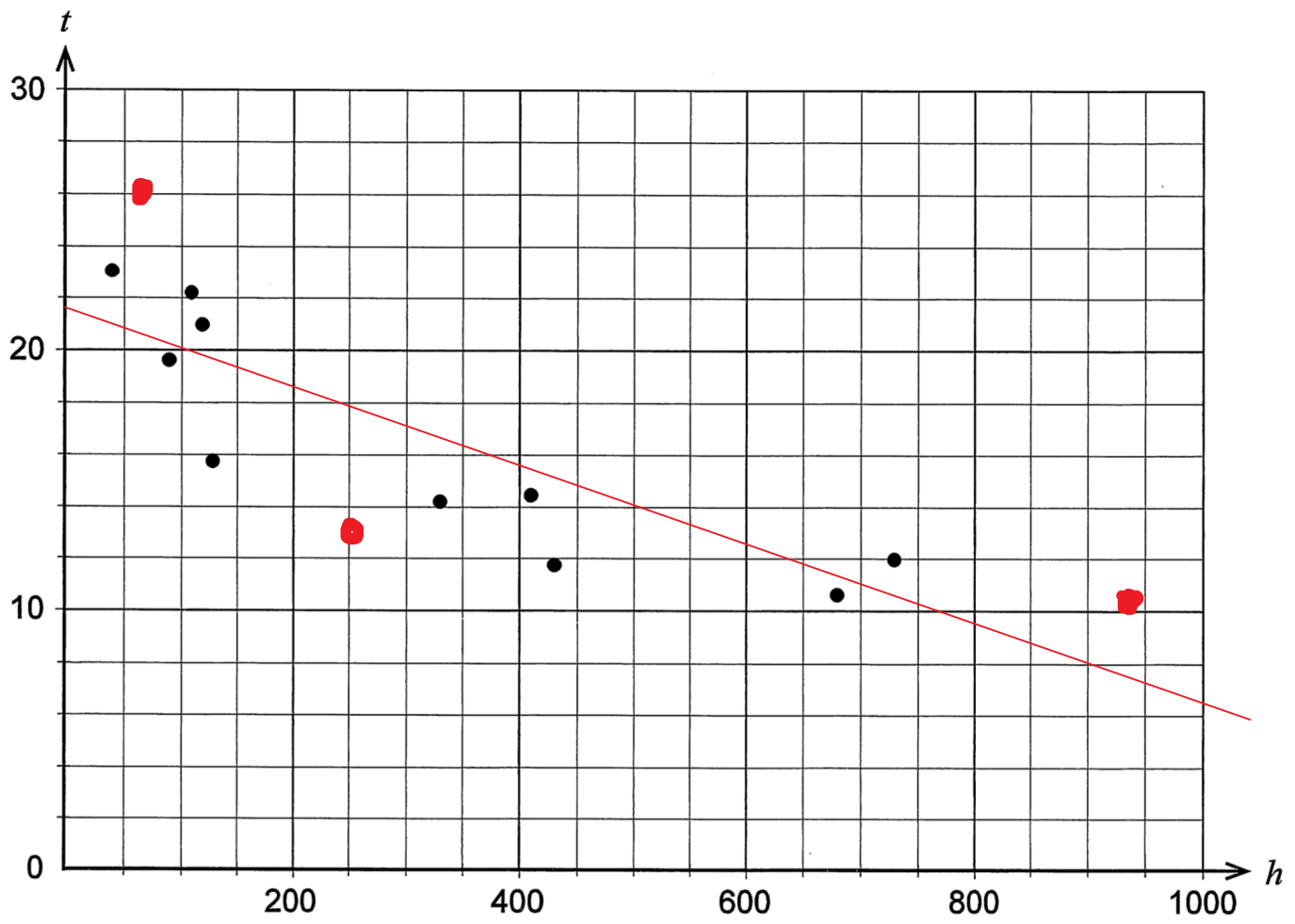
(d) Due to some unforeseen problems with Activities G and J, **one** of these activities will require an extra three hours to complete. Which of the activities should be chosen for the completion time to be at a minimum? Justify your answer. (3 marks)

Choose G: J has only 1 hour of float time whereas G has 2, so project will only be delayed by 1 hour.

Q12ab

Question 12 **(8 marks)**

The Bureau of Meteorology recorded data taken from several weather stations. The scatterplot below shows the height, h (m), of each weather station above sea level and the mean minimum temperature, t (°C), recorded at that station for the month of April.



The following table provides information for three more weather stations for the month of April.

Height of weather station above sea level, h (m)	250	60	930
Mean minimum temperature, t (°C)	13.1	26.2	10.6

(a) Plot this additional information on the scatterplot above. **(2 marks)**

Another bivariate data question not suitable for fitting linear model…

(b) The equation of the least-squares line for these data is $t = -0.015h + 21.476$. Draw this line on the scatterplot above. **(2 marks)**

```
21.476−0.015×0
                    21.476
21.476−0.015×1000
                    6.476
```

Q12cde

(c) The correlation coefficient (r) was determined for the collected data. Circle the value of r most likely to be the result from the list below. (1 mark)

$r = -1.2$ ✗

$r = -0.8$ ✓

$r = -0.2$ ✗

$r = 0.5$ ✗

$r = 0.9$ ✗

(d) Identify whether the nature of the relationship between the height of a weather station above sea level, h, and the mean minimum temperature, t (°C), is linear or non-linear. (1 mark)

Non-linear

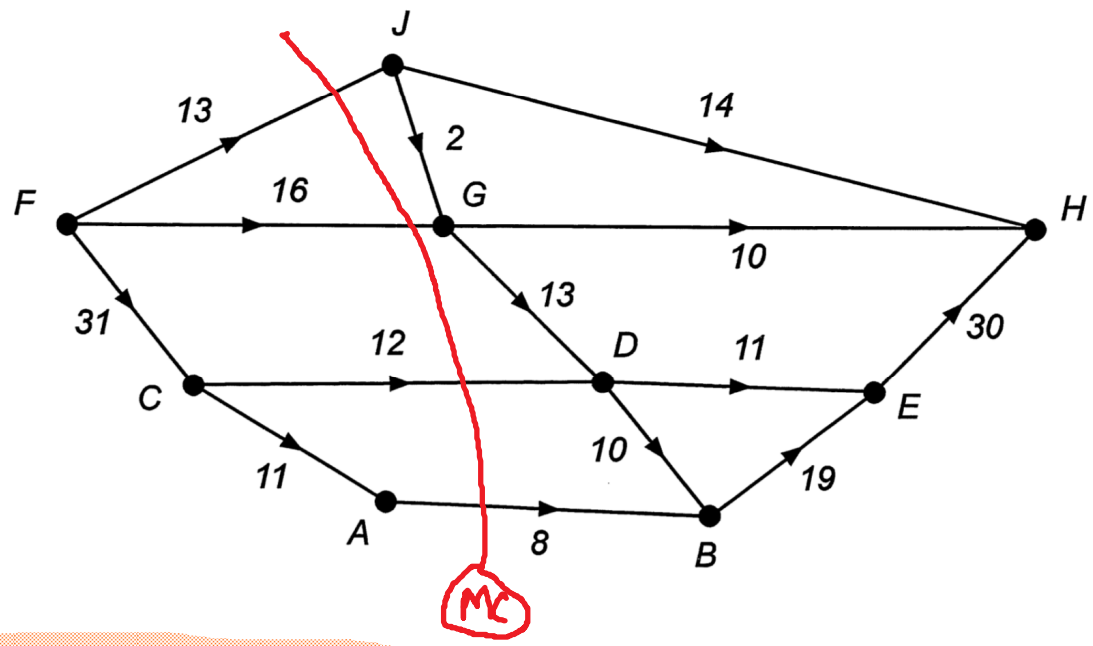
(e) A spokesperson for the Bureau of Meteorology summarised the information from parts (a)–(d), saying 'It is evident that raising the height of a weather station above sea level causes the mean minimum temperature to drop'. Is this statement correct? Justify your decision. (2 marks)

No. The spokesperson has provided no evidence to support causation, merely observations that suggest a relationship.

Q13

Question 13 **(8 marks)**

The traffic flow (in hundreds of cars per hour) through a road network (F to H) is shown below.



(a) By listing the different paths and their flow rate, determine the maximum flow through the network. **(4 marks)**

FJH: 13
FGH: 10
FGDEH: 6
FCDEH: 4
FCDBEH: 8
FCABEH: 8
Total 49 hundred cars per hour

(b) Verify the maximum flow obtained in part (a) by showing the minimum cut on the given network. **(1 mark)**

(c) (i) If **one** road is to be widened to allow for more traffic, which road should be chosen to increase the maximum flow the most? **(1 mark)**

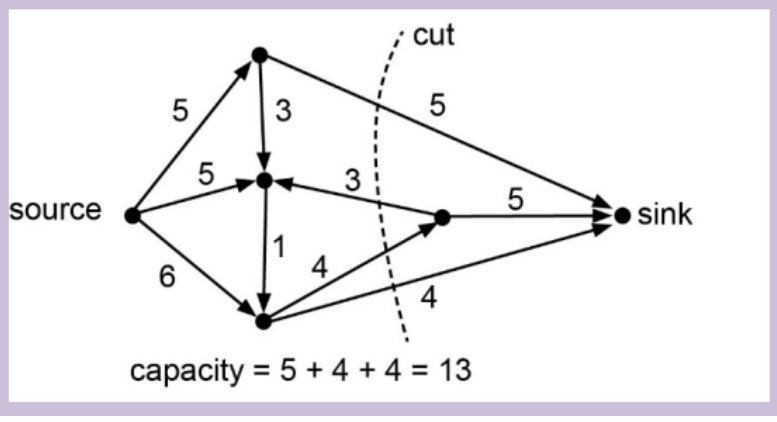
AB

(ii) How much more traffic should this road allow to flow and what would be the new maximum flow for the network? **(2 marks)**

An extra 300 cars/h, so new max flow 5 200 cars/h

Flow networks

4.3.9 solve small-scale network flow problems, including the use of the 'maximum flow-minimum cut' theorem

Cut (in a flow network)	In a flow network, a cut is a partition of the vertices of a graph into two separate groups, with the source in one group and the sink in the other. The capacity of the cut is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored. Example: 
Flow network	A flow network is a directed graph where each edge has a capacity (for example, 100 cars per hour, 800 litres per minute) and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a source, which has more outgoing flow, or a sink, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes. Small-scale network problems could, for example, include determining the maximum volume of oil flow through a network of pipes from an oil stoorage tank (the source) to a terminal (the sink).

Q14

Question 14 **(13 marks)**

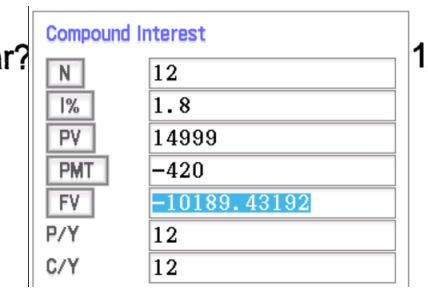
Andrew takes out a $14 999 loan to purchase his first car after paying a $1200 deposit. The car dealer offered the loan at an introductory interest rate of 1.80% p.a. for the first year and then the rate becomes 3.24% p.a. for the remaining time of the loan. Interest is added monthly and Andrew has calculated he can afford to make monthly repayments of $420.

(a) (i) Express the loan repayment process for the first year as a recursive formula. (2 marks)

$$T_{n+1} = T_n \times 1.0015 - 420, T_0 = 14999$$

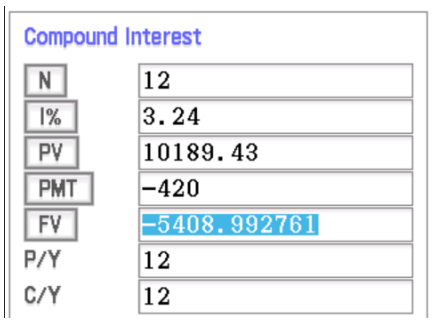
(ii) How much does Andrew still owe after one year? (1 mark)

$10 189.43



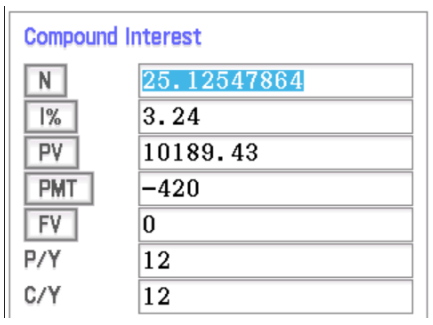
(b) How much does Andrew owe after two years? (3 marks)

$5 408.99



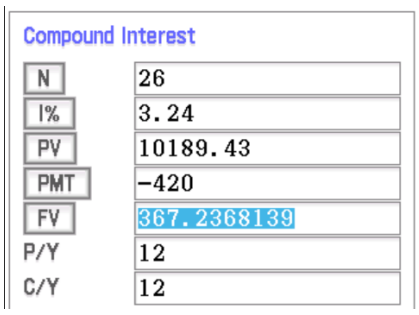
(c) How long does it take Andrew to repay the loan? (2 marks)

$12 + 26 = 38$ months



(d) Determine the amount of the final repayment. (2 marks)

$\$420 - 367.24 = \$52.76$



(e) Calculate the total cost of the car. (3 marks)

$1200 + 37 \times 420 + 52.76 = \$16\ 792.76$

Q15a

Question 15 **(15 marks)**

(a) The table below shows some time series data where t represents time.

t	1	2	3	4	5	6	7	8
x	14	17	18	24	21	19	16	13

Calculate at $t = 4$

(i) the 3-point moving average. (1 mark)

21

```
(18+24+21)/3
                    21
(7+17+18+24+21+19+8)/6
                    19
```

(ii) the 6-point centred moving average. (2 marks)

19

(b) A retailer in a shopping centre sells mobile phones. The data of its quarterly sales, together with some calculations, are shown in the table below.

Year	Data number (n)	Quarter	Mobile phone sales	Quarterly mean	Percentage of quarterly mean	Deseasonalised figure (D)
2013	1	March	901	905	99.56	915
	2	June	802		88.62	914
	3	September	A		97.68	900
	4	December	1033		114.14	894
2014	5	March	973	984.5	98.83	988
	6	June	863		C	984
	7	September	964		97.92	981
	8	December	1138		115.59	985
2015	9	March	1049	1065.5	98.45	1065
	10	June	932		87.47	$\div 51 = E$
	11	September	1049		98.45	1068
	12	December	1232		115.63	1066
2016	13	March	1119	B	97.01	1136
	14	June	1006		87.21	1147
	15	September	1142		99.00	1162
	16	December	1347		116.78	1166

Q15b

(i) Determine the value of A, B and C in the table in part (b) on the previous page. (3 marks)

$A = 884$
$B = 1153.5$
$C = 87.66$

```
4×905−901−802−1033
                  884
(1119+1006+1142+1347)/4
                1153.5
863/984.5
          0.8765871001
```

(ii) Complete the Seasonal Index table below. (1 mark)

Quarter	March	June	September	December
Seasonal Index	0.9846	0.8774	0.9826	1.1554

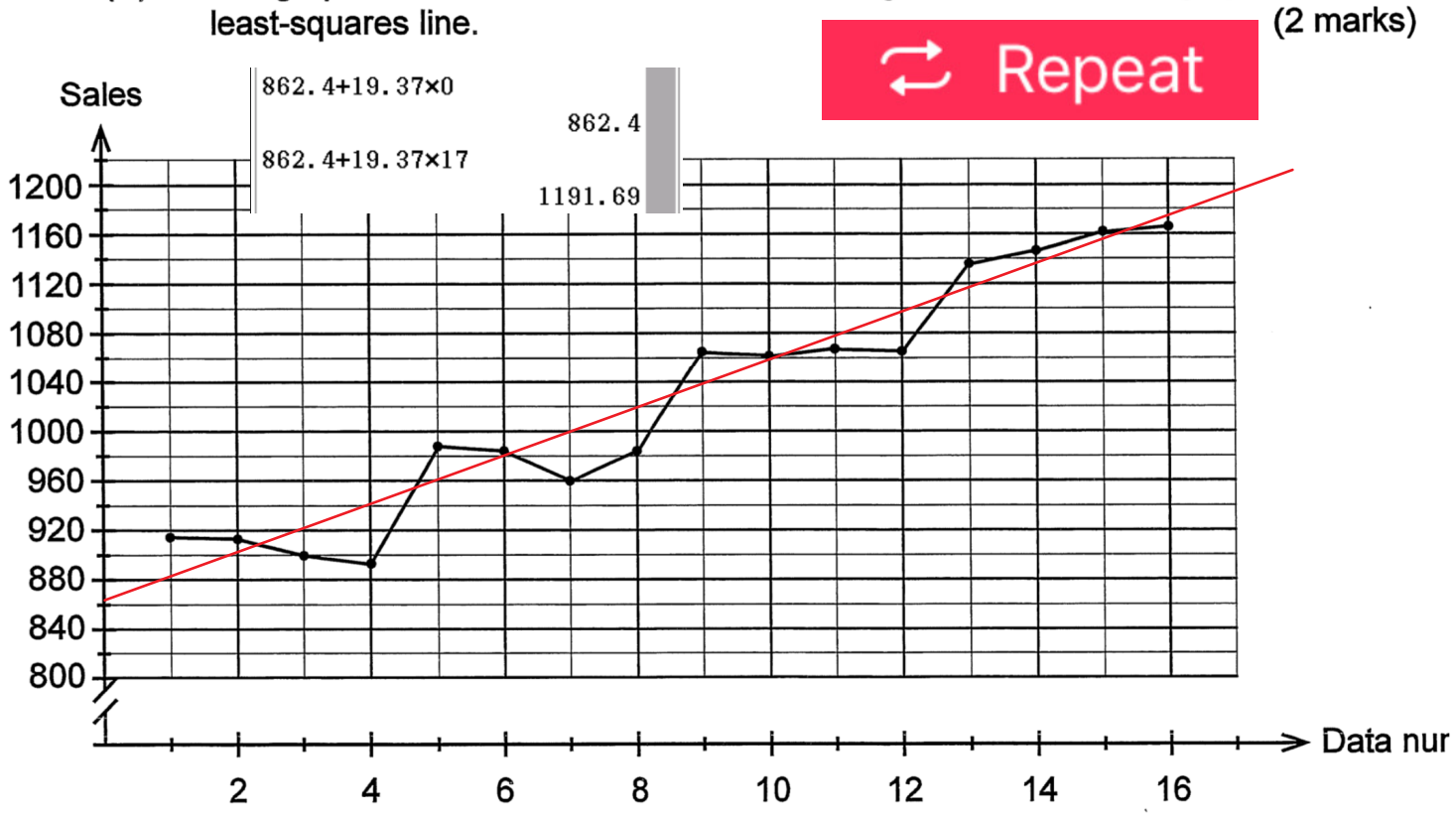
(iii) Determine the value of E in the table in part (b) on the previous page. (2 marks)

$E = 1062$

```
4−0.9846−0.8774−0.9826
                1.1554
932/0.8774
          1062.229314
```

The equation of the least-squares line for deseasonalised figure against data number is $D = 19.37n + 862.4$.

(iv) The graph below shows the deseasonalised figures. Draw, on the graph, the least-squares line. (2 marks)



(v) Predict the mobile phone sales for December 2017. (2 marks)

$n = 20$
$D = 1249.8 \times 1.1554 = 1444$

```
862.4+19.37×20
                1249.8
ans×1.1554
          1444.01892
```

(vi) Comment on the reliability of your prediction made in part (v). (2 marks)

Assuming that the trend and seasonality continue, prediction is reliable as strong relationship apparent and only extrapolating within next cycle of data.

3AB Examiners report

Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Report on the 2014 WACE examination in
Mathematics 3A/3B

Question 14 Attempted by 3387 candidates Mean 7.44(/12) 62% Max 12 Min 0

Part (a) was done fairly well. However, '4' was given as the most common incorrect answer, rather than quarterly. In part (b) most candidates answered this part well with most errors occurring in the calculation of A. Part (c) was well done generally, apart from some candidates carelessly leaving out the minus sign. The most common error was forgetting to add the seasonal component in part (d). In part (e) most candidates stated it was unreliable due to extrapolation. Candidates seem confused about the concept of extrapolation used in time-series data. In part (f) the most common error was adding the seasonal component to D.

MATHEMATICS 3A/3B **10** **CALCULATOR-ASSUMED**

Question 14 **(12 marks)**

The table below shows the cost of gas for a Perth household. Accounts are sent at the end of each time period. Accounts for 2010 are unavailable, but some statistics are provided.

Time (t)	Months/Year	Cost ($)	Four-point centred moving averages (m)	Residual
1	November 2009–January 2010	D	-	-
2	February 2010–April 2010	-	-	-
3	May 2010–July 2010	-	-	-
4	August 2010–October 2010	A	-	-
5	November 2010–January 2011	121.52	138.84	–17.32
6	February 2011–April 2011	140.14	139.07	1.08

(e) Comment on the reliability of your prediction from part (d). (2 marks)

Solution
The prediction is fairly reliable as it is within one cycle of the data.
Specific behaviours
✓ correctly states that the prediction is fairly reliable ✓ correctly gives a reason for this statement

Q16

Question 16 **(8 marks)**

In a Northern Territory river, the crocodile population is dropping by 7.5% each year. The current population is 200. A scheme is being trialled under which 20 crocodiles are introduced to the river each year.

The population of crocodiles in the river can be modelled by the first-order linear recurrence relation $T_{n+1} = 0.925T_n + b$, $T_1 = 200$, where T_n is the number of crocodiles in the river at the beginning of the n^{th} year.

(a) (i) Interpret the coefficient 0.925 in the context of the question. (1 mark)

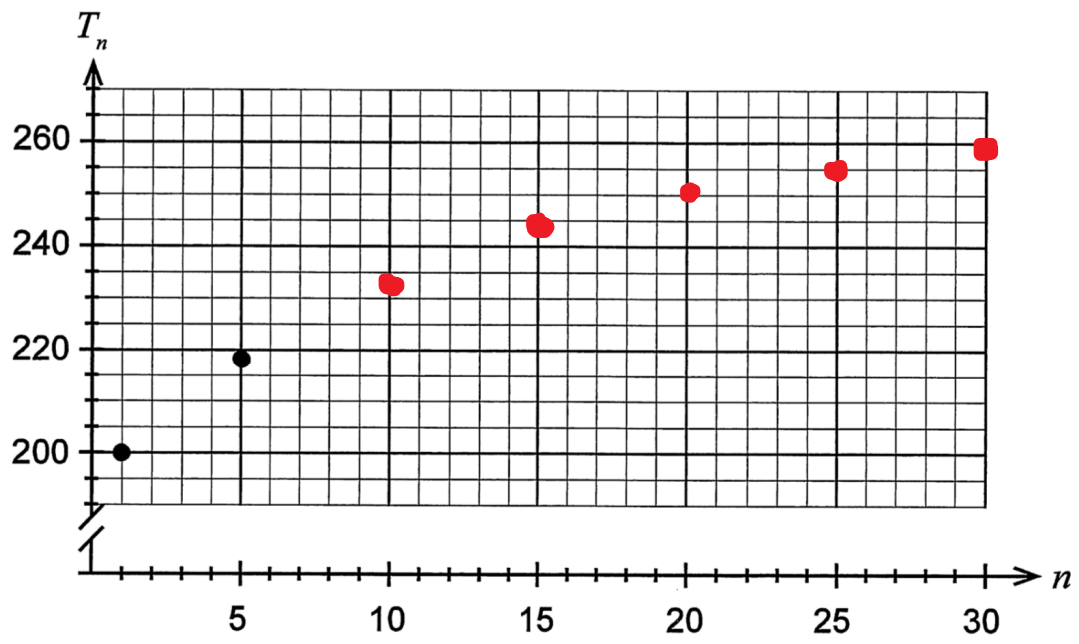
Just 92.5% of the population survives each year (100-7.5)

(ii) State the value of b. (1 mark)

$b = 20$

$a_{n+1}=a_n \cdot 0.925+20$	
n	a_n
9	230.94
10	233.62
11	236.09

(b) Graph the number of crocodiles in the river for every five year period (commencing at $n = 5$), up to the 30th year on the axes below. (2 marks)



(c) Using your graph, comment on how the population of crocodiles is changing over time. (2 marks)

Increasing at a decreasing rate

(d) To the nearest whole number, what is the long-term effect on the crocodile population? (2 marks)

267 crocodiles

96	266.63
97	266.63
98	266.63
99	266.63
100	266.64

solve(x=0.925x+20
{x=266.6666667}